Proof Writing Practice

Definitions: An integer is even if it has the form $2n$ for some $n \in \mathbb{Z}$. An integer is *fun* if it has the form $3m$ for some $m \in \mathbb{Z}$. An integer is *happy* if it has the form $6k$ for some $k \in \mathbb{Z}$.

Theorem. An even integer times a fun integer is happy.

Proof.

Question: How do you write an assertion of the form $\mathcal{A} \subseteq \mathcal{B}$ into an assertion of the form "if p then q"?

e.g. Theorem. If $\mathcal{A} = \{8k + 3 \mid k \in \mathbb{Z}\}$ and $\mathcal{B} = \{4k + 7 \mid k \in \mathbb{Z}\}$, then $\mathcal{A} \subseteq \mathcal{B}$.

Proof.

Question: How do you write an assertion of the form $\mathcal{S} = \mathcal{T}$ into an assertion of the form "if p then q"?

e.g. Theorem. If $\mathcal{S} = \{6m + 3 \mid m \in \mathbb{Z}\}$ and $\mathcal{T} = \{6\ell + 15 \mid \ell \in \mathbb{Z}\}$, then $\mathcal{S} = \mathcal{T}$.

Proof.

Question: When does this contrapositive stuff become useful?

e.g. Theorem. Let $x, y \in \mathbb{Z}$. If $x + y$ is even, then x and y have the same parity.

(Before we start, let's think about the definitions of *even* and *odd*.)

Proof.