

Archimedes' principle

It is named after Archimedes of Syracuse, who first discovered this law. According to Archimedes' principle, "Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Vitruvius (De architectura IX.9–12) recounts the famous story of Archimedes making this discovery while in the bath. He was given the task of finding out if a goldsmith, who worked for the king, was carefully replacing the king's gold with silver. While doing this Archimedes decided he should take a break so went to take a bath. While entering the bath he noticed that when he placed his legs in, water spilled over the edge. Struck by a moment of realisation, he shouted "Eureka!" He informed the king that there was a way to positively tell if the smith was cheating him. Knowing that gold has a higher density than silver, he placed the king's crown and a gold crown of equal weight into a pool. Since the king's crown caused more water to overflow, it was, therefore, less dense, Archimedes concluded that it contained silver, causing the smith to be executed. The actual record of Archimedes' discoveries appears in his two-volume work, *On Floating Bodies*. The ancient Chinese child prodigy Cao Chong (196–208 AD) also applied the principle of buoyancy in order to accurately weigh an elephant, as described in the Sanguo Zhi, also known as the Records of Three Kingdoms.

Archimedes' principle does not consider the surface tension (capillarity) acting on the body.^[1]

The weight of the displaced fluid is directly proportional to the volume of the displaced fluid (if the surrounding fluid is of uniform density). Thus, among completely submerged objects with equal masses, objects with greater volume have greater buoyancy.

Suppose a rock's weight is measured as 10 newtons when suspended by a string in a vacuum. Suppose that when the rock is lowered by the string into water, it displaces water of weight 3 newtons. The force it then exerts on the string from which it hangs would be 10 newtons minus the 3 newtons of buoyant force: $10 - 3 = 7$ newtons. Buoyancy reduces the apparent weight of objects that have sunk completely to the sea floor. It is generally easier to lift an object up through the water than it is to pull it out of the water.

The density of the immersed object relative to the density of the fluid can easily be calculated without measuring any volumes:

$$\frac{\text{Density of object}}{\text{Density of fluid}} = \frac{\text{weight}}{\text{weight} - \text{apparent immersed weight}}$$

<http://en.wikipedia.org/wiki/Buoyancy>

Density has many applications in the chemical industry. The relationship between mass and volume is an important aspect of the specification and utilization of both solids and liquids. For example, valuable metals and stones are characterized by their densities. Bulk chemicals are shipped in drums and totes, most often by the pound. Conversion of pounds to gallons or into metric equivalents is a critical aspect of trade. Shipping costs are most often determined by weight. Density can be used to quantify the dissolved solids in liquids. For example, high concentrations of salt in brines increase the density of solutions. The most common units for density are g/mL (g/cm^3), or pounds per gallon.

Specific gravity related to density, but is a unitless quantity, defined as the density of a substance divided by the density of water. Since we often assume the density of water to be 1.0 g/mL, the specific gravity usually agrees closely with density. However, as temperature changes, so does the density of water, so at elevated temperatures, specific gravity can be somewhat different than density of the material being tested. Most often in the lab, a fixed volume container called a pycnometer is used to determine specific gravity. The pycnometer is filled with water and weighed. Then, the pycnometer is filled with the unknown liquid and weighed. The mass of the unknown liquid divided by the mass of the water is the specific gravity.

Measurement of the density of an unknown solid is relatively easy. Determine both the mass and the volume of a substance, and then divide mass by the volume to calculate density. Archimedes discovered that volumes can be measured by displaced volumes of water. He also discovered that a solid mass weighs more in air than when suspended in water. The difference in the two masses is the mass of the displaced liquid. When using water as the liquid, the difference in grams is approximately the same as milliliters. So, by dividing the mass of the solid in air by the *difference* between its mass in air and its mass suspended in water, the density is obtained. Of course, if the liquid is not water, then the density of the liquid must be taken into account and the density of the liquid must be used to convert the displaced mass into displaced volume before the density of the suspended solid can be calculated.

$$\rho_{\text{solid}} = \frac{m_{\text{air}}}{m_{\text{air}} - m_{\text{liquid}}} (\rho_{\text{refliq}})$$

Where ρ_{solid} is the density of the solid to be measured, m_{air} is the mass of the solid in air, and m_{liquid} is the mass of the solid while suspended in liquid. The ρ_{refliq} is the density of the reference liquid at the temperature during the analysis. (*Data available in tables for water and alcohol.*)

An interesting application of this same equation is the determination of the density of a liquid by weighing a suspended solid of known mass and volume in the liquid. (A known, calibrated solid used for this purpose is often called a "sinker." How might this determination of density of the liquid be obtained? Consider this following equation:

$$\rho_{\text{unkliq}} = \frac{m_{\text{displaced liquid}}}{V_{\text{sinker}}}$$

Where ρ_{unkliq} is the density of the liquid to be measured, $m_{\text{displaced liquid}}$ is the mass of the displaced liquid, and V_{sinker} is the volume of the sinker. So, the density of a liquid may be determined by weighing a sinker immersed in the liquid, if the exact volume of the sinker is known.

Question: How would the volume of the sinker affect the reproducibility of the determination? Is it better to use a large sinker or a smaller one? Explain.

Formulae for determining the density of solids with compensation for air density

$$\rho = \frac{A}{A-B} (\rho_0 - \rho_L) + \rho_L$$

$$V = \alpha \frac{A - B}{\rho_0 - \rho_L}$$

- ρ = Density of the sample
 A = Weight of the sample in air
 B = Weight of the sample in the auxiliary liquid
 V = Volume of the sample
 ρ_0 = Density of the auxiliary liquid
 ρ_L = Density of Air (0.0012 g/cm³)
 α = Weight correction factor (0.99985), to take the atmospheric buoyancy of the adjustment weight into account

Formula for determining the density of liquids with compensation for air density

$$\rho = \alpha \frac{P}{V} + \rho_L$$

- ρ = Density of the liquid
 P = Weight of the displaced liquid
 V = Volume of the sinker
 ρ_L = Density of air (0.0012 g/cm³)
 α = Weight correction factor (0.99985), to take the atmospheric buoyancy of the adjustment weight into account

Density Table for Distilled Water

T/°C	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10.	0.99973	0.99972	0.99971	0.99970	0.99969	0.99968	0.99967	0.99966	0.99965	0.99964
11.	0.99963	0.99962	0.99961	0.99960	0.99959	0.99958	0.99957	0.99956	0.99955	0.99954
12.	0.99953	0.99951	0.99950	0.99949	0.99948	0.99947	0.99946	0.99944	0.99943	0.99942
13.	0.99941	0.99939	0.99938	0.99937	0.99935	0.99934	0.99933	0.99931	0.99930	0.99929
14.	0.99927	0.99926	0.99924	0.99923	0.99922	0.99920	0.99919	0.99917	0.99916	0.99914
15.	0.99913	0.99911	0.99910	0.99908	0.99907	0.99905	0.99904	0.99902	0.99900	0.99899
16.	0.99897	0.99896	0.99894	0.99892	0.99891	0.99889	0.99887	0.99885	0.99884	0.99882
17.	0.99880	0.99879	0.99877	0.99875	0.99873	0.99871	0.99870	0.99868	0.99866	0.99864
18.	0.99862	0.99860	0.99859	0.99857	0.99855	0.99853	0.99851	0.99849	0.99847	0.99845
19.	0.99843	0.99841	0.99839	0.99837	0.99835	0.99833	0.99831	0.99829	0.99827	0.99825
20.	0.99823	0.99821	0.99819	0.99817	0.99815	0.99813	0.99811	0.99808	0.99806	0.99804
21.	0.99802	0.99800	0.99798	0.99795	0.99793	0.99791	0.99789	0.99786	0.99784	0.99782
22.	0.99780	0.99777	0.99775	0.99773	0.99771	0.99768	0.99766	0.99764	0.99761	0.99759
23.	0.99756	0.99754	0.99752	0.99749	0.99747	0.99744	0.99742	0.99740	0.99737	0.99735
24.	0.99732	0.99730	0.99727	0.99725	0.99722	0.99720	0.99717	0.99715	0.99712	0.99710
25.	0.99707	0.99704	0.99702	0.99699	0.99697	0.99694	0.99691	0.99689	0.99686	0.99684
26.	0.99681	0.99678	0.99676	0.99673	0.99670	0.99668	0.99665	0.99662	0.99659	0.99657
27.	0.99654	0.99651	0.99648	0.99646	0.99643	0.99640	0.99637	0.99634	0.99632	0.99629
28.	0.99626	0.99623	0.99620	0.99617	0.99614	0.99612	0.99609	0.99606	0.99603	0.99600
29.	0.99597	0.99594	0.99591	0.99588	0.99585	0.99582	0.99579	0.99576	0.99573	0.99570
30.	0.99567	0.99564	0.99561	0.99558	0.99555	0.99552	0.99549	0.99546	0.99543	0.99540

Density Table for Ethanol

T/°C	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10.	0.79784	0.79775	0.79767	0.79758	0.79750	0.79741	0.79733	0.79725	0.79716	0.79708
11.	0.79699	0.79691	0.79682	0.79674	0.79665	0.79657	0.79648	0.79640	0.79631	0.79623
12.	0.79614	0.79606	0.79598	0.79589	0.79581	0.79572	0.79564	0.79555	0.79547	0.79538
13.	0.79530	0.79521	0.79513	0.79504	0.79496	0.79487	0.79479	0.79470	0.79462	0.79453
14.	0.79445	0.79436	0.79428	0.79419	0.79411	0.79402	0.79394	0.79385	0.79377	0.79368
15.	0.79360	0.79352	0.79343	0.79335	0.79326	0.79318	0.79309	0.79301	0.79292	0.79284
16.	0.79275	0.79267	0.79258	0.79250	0.79241	0.79232	0.79224	0.79215	0.79207	0.79198
17.	0.79190	0.79181	0.79173	0.79164	0.79156	0.79147	0.79139	0.79130	0.79122	0.79113
18.	0.79105	0.79096	0.79088	0.79079	0.79071	0.79062	0.79054	0.79045	0.79037	0.79028
19.	0.79020	0.79011	0.79002	0.78994	0.78985	0.78977	0.78968	0.78960	0.78951	0.78943
20.	0.78934	0.78926	0.78917	0.78909	0.78900	0.78892	0.78883	0.78874	0.78866	0.78857
21.	0.78849	0.78840	0.78832	0.78823	0.78815	0.78806	0.78797	0.78789	0.78780	0.78772
22.	0.78763	0.78755	0.78746	0.78738	0.78729	0.78720	0.78712	0.78703	0.78695	0.78686
23.	0.78678	0.78669	0.78660	0.78652	0.78643	0.78635	0.78626	0.78618	0.78609	0.78600
24.	0.78592	0.78583	0.78575	0.78566	0.78558	0.78549	0.78540	0.78532	0.78523	0.78515
25.	0.78506	0.78497	0.78489	0.78480	0.78472	0.78463	0.78454	0.78446	0.78437	0.78429
26.	0.78420	0.78411	0.78403	0.78394	0.78386	0.78377	0.78368	0.78360	0.78351	0.78343
27.	0.78334	0.78325	0.78317	0.78308	0.78299	0.78291	0.78282	0.78274	0.78265	0.78256
28.	0.78248	0.78239	0.78230	0.78222	0.78213	0.78205	0.78196	0.78187	0.78179	0.78170
29.	0.78161	0.78153	0.78144	0.78136	0.78127	0.78118	0.78110	0.78101	0.78092	0.78084
30.	0.78075	0.78066	0.78058	0.78049	0.78040	0.78032	0.78023	0.78014	0.78006	0.77997

Density of C₂H₅OH according to the "American Institute of Physics Handbook".

Densities of Common Metals

Source: <http://www.coolmagnetman.com/magconda.htm>

Density (Measured in Various Units)

metal	g/cm ³	lb/in ³	lb/ft ³	lb/gal
water	1.00	0.036	62	8.35
aluminum	2.7	0.098	169	22.53
zinc	7.13	0.258	445	59.5
iron	7.87	0.284	491	65.68
copper	8.96	0.324	559	74.78
silver	10.49	0.379	655	87.54
lead	11.36	0.41	709	94.8
mercury	13.55	0.49	846	113.08
gold	19.32	0.698	1206	161.23

Specific Gravities (Density of Metal/Density of Water)

water	1.00	1.00	1.00	1.00
aluminum	2.70	2.72	2.73	2.70
zinc	7.13	7.17	7.18	7.13
iron	7.87	7.89	7.92	7.87
copper	8.96	9.00	9.02	8.96
silver	10.49	10.53	10.56	10.48
lead	11.36	11.39	11.44	11.35
mercury	13.55	13.61	13.65	13.54
gold	19.32	19.39	19.45	19.31

PROPERTIES OF SODIUM CHLORIDE SOLUTIONS (60°F)

Source: <http://texasbrine.com/tables/properties.html>

% Sodium Chloride	Specific	Weight	mg/l*	mg/l*	Freezing
by weight	Gravity	lbs. per Gallon	Sodium Chloride	Chloride	Point ° F
1.0	1.007	8.40	10,070	6,110	31.0
2.0	1.014	8.46	20,280	12,300	30.0
3.0	1.021	8.52	30,630	18,580	28.9
4.0	1.029	8.59	41,160	24,970	27.8
5.0	1.036	8.65	51,800	31,420	26.7
6.0	1.043	8.70	62,580	37,960	25.5
7.0	1.051	8.77	73,570	44,630	24.2
8.0	1.059	8.84	84,720	51,390	22.9
9.0	1.067	8.90	96,030	58,250	21.6
10.0	1.074	8.96	107,400	65,150	20.2
11.0	1.082	9.03	119,020	72,200	18.8
12.0	1.089	9.09	130,680	79,270	17.3
13.0	1.097	9.15	142,610	86,510	15.7
14.0	1.104	9.21	154,560	93,760	14.1
15.0	1.112	9.28	166,800	101,180	12.4
16.0	1.119	9.34	179,040	108,610	10.6
17.0	1.127	9.41	191,590	116,220	8.7
18.0	1.135	9.47	204,300	123,930	6.7
19.0	1.143	9.54	217,170	131,740	4.6
20.0	1.151	9.61	230,200	139,640	2.4
21.0	1.159	9.67	243,390	147,650	0.0
22.0	1.168	9.75	256,960	155,880	-2.5
23.0	1.176	9.81	270,480	164,080	-5.2
24.0	1.184	9.88	284,160	172,380	1.4
25.0	1.193	9.96	298,250	180,920	13.3
26.0	1.201	10.02	312,260	189,420	27.9

*milligrams per liter (mg/l) may be converted to parts by million (ppm) by dividing mg/l by the specific gravity times 10,000.

The relationship between mg/l sodium chloride and mg/l chloride is shown as follows:

mg/l Sodium Chloride = mg/l Chloride x 1.649

mg/l Chloride = mg/l Sodium Chloride x 0.6066