Effects of Multilevel Versus Unilevel Metacognitive Training on Mathematical Reasoning

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ABSTRACT  The effects of 3 instructional methods on mathematical reasoning were investigated. The methods are (a) cooperative learning embedded within multilevel metacognitive training (MMT), (b) cooperative learning embedded within unilevel metacognitive training (UMT), and (c) learning in the whole class with no metacognitive training. MMT was implemented in mathematics and English classrooms; UMT was used only in mathematics classrooms; and the whole class with no metacognitive training served as a control group. Results indicated that students who were exposed to MMT significantly outperformed their counterparts who were exposed to UMT who, in turn, significantly outperformed the control group. Effects of MMT were observed on students while they solved mathematical problems. Theoretical and practical implications of the study are discussed.

Key words: authentic tasks, cooperative learning, metacognitive training

For more than a decade, metacognition researchers have sought instructional methods that use metacognitive processes to enhance mathematical reasoning. In particular, King (1991, 1994) and Mevarech and Kramarski (1997) suggested structuring group interaction to provide metacognitive training that focuses on students' understanding of the task, on awareness and self-regulation of strategy application, and on constructing connections between prior and new knowledge.

Mevarech and Kramarski's (1997) method, IMPROVE, is an example of metacognitive instruction that emphasizes reflective discourse by providing each student with the opportunity to be involved in mathematical reasoning via the use of metacognitive questions. The questions emphasize the following: (a) nature of the problem (e.g., What is the problem all about?); (b) use of strategies appropriate for solving the problem (e.g., What are the strategies/tactics/principles that are appropriate to solve the problem, and why?); and (c) construction of relationships between previous and new knowledge (e.g., What are the similarities/differences between the problem at hand and the problems solved in the past?).

IMPROVE represents all the teaching/learning stages of the metacognitive training: Introducing the new topics to the whole class; Metacognitive questioning in small groups; Practicing, Reviewing, and Obtaining mastery on higher and lower cognitive skills; and Verifying and Enriching. Mevarech and Kramarski (1997) reported that students who were exposed to IMPROVE outperformed their counterparts in the nontreatment control groups. In particular, we observed the positive effects of IMPROVE on problem solving that requires higher order skills. Further results on IMPROVE showed that, within cooperative settings, students who were exposed to the metacognitive training significantly outperformed their counterparts who were not exposed to such training, regarding their ability to explain mathematical reasoning in writing and to activate metacognitive processes in small groups (Mevarech & Kramarski, 1997, in press).

The contribution of metacognitive training has been documented not only with regard to mathematical reasoning but also with regard to reading comprehension (King, 1991, 1994; Pressley, 1986; Salomon, Globerson, & Guterman, 1989). Pressley, for example, emphasized the effectiveness of reflecting on reading strategies, whereas King (1991, 1994) advocated bridging existing and new knowledge to improve reading comprehension. Salomon et al. reported that students who were exposed to metacognitive instruction in reading classrooms improved not only their reading comprehension skills but also their writing ability assessed as a transfer task. Given those studies, we hypothesized that providing metacognitive training in both mathematics and English classrooms (MMT) would exert more positive effects on students' achievement than would implementing metacognitive training in only mathematics or English classrooms (UMT), not only because the provision of a "double portion" of metacognitive training is assumed to be more effective than "one portion" but also

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because of the possibility that students who are exposed to MMT may be able to generalize the use of metacognitive processes beyond a specific domain. Thus, the effects of MMT are expected to be observed on mathematical reasoning, the activation of metacognitive processes, and the solution of a transfer task that focuses on authentic, real-life problems.

Therefore, the purpose of the present research is threefold: (a) to investigate mathematical reasoning of students who were exposed to either multilevel, unilevel, or no metacognitive training; (b) to compare the differential effects of these methods on various aspects of the solution of a transfer task on the basis of a real-life situation; and (c) to examine the differential effects of the methods on students’ metacognitive knowledge.

Method

Participants

Participants were 182 students (95 boys and 87 girls) who studied in 6 seventh-grade classrooms randomly selected from three junior high schools—two classes from each school. We selected the three schools from a pool of junior high schools in which mathematics was taught in heterogeneous classrooms with no grouping or ability tracking. Each integrated junior high school included students from different socioeconomic status as defined by the Israel Ministry of Education. The classes were similar in size, and students’ mean age was 12.4 years. Levels of mathematics achievement were assessed prior to the beginning of this study.

Each of the six teachers who participated in this study taught one classroom. All the teachers were women who had a similar level of education (B.Ed. major in mathematics), had more than 5 years of experience in teaching mathematics, and had taught in heterogeneous classrooms. The teachers were exposed to a 1-day in-service training as described in the Treatment section. We assigned schools randomly to one of the following conditions:

1. MMT: Students studied both mathematics and English with the IMPROVE method \((n = 60)\).
2. UMT: Students studied only mathematics with the IMPROVE method \((n = 60)\).
3. Control group: Students did not study with the IMPROVE method; that is, students were not directly exposed to metacognitive training \((n = 62)\).

Treatment

Under all conditions, each period in the in-service teacher training included three parts: (a) introduction to the whole class (about 10 min); (b) activities practice (about 30 min); and (c) review with the whole class (about 5 min).

The differences between treatments were as follows:

**MMT group:** In this condition, the teacher implemented the IMPROVE method (Mevarech & Kramarski, 1997) in both mathematics and English (as a foreign language) classes. A discussion was held in the mathematics class on the similarities and differences in learning a foreign language (English) and a mathematical language (Pimm, 1987, 1996). The teacher presented the stages of IMPROVE and held a discussion on the importance of using metacognitive questions in both mathematics and English classrooms. In addition, students discussed the common factors of mathematics language and English, including the importance of understanding vocabulary and symbols, using appropriate strategies to read a text or solve a problem, and reflecting on the text or the solution.

Each session (in mathematics and English classrooms) began with the teacher’s short presentation of the new materials to the whole class using metacognitive self-addressed questioning techniques according to comprehension, connection, and strategy questions (Mevarech & Kramarski, 1997). For example, students used the following metacognitive questions in mathematics lessons: What is the problem/task all about? How is this problem/task different from what you have already solved? Which strategy/principle is appropriate for solving/addressing the problem/task? The students used similar questions in English lessons: What am I supposed to do in this task? What do I already know about the story/task? What are the differences/similarities between the texts? Which strategy is appropriate for understanding the text?

Following the introduction, students started to work in small groups using the materials that we designed. Students studied in heterogeneous groups comprised of 4 members: 1 high-achieving student, 2 average-achieving students, and 1 low-achieving student. The students studied in groups as follows: each student, by turn, read a problem/text aloud and tried to solve or analyze it. Whenever there was no consensus, the group discussed the issue until the disagreement was resolved. Students were encouraged to talk about the problem/text, explain it to each other, and approach it from different perspectives. Students used the metacognitive questions during their discourse in small-group activities and in their written explanations when they solved the mathematical problems or addressed the texts in the English class. Periodically, the teachers bridged the metacognitive processes that were activated in the mathematics and English classrooms.

**UMT group:** In this condition, students were exposed to IMPROVE only in mathematics classrooms. The teacher presented the stages of IMPROVE to the students and discussed the importance of using the method in mathematics classrooms. Students studied in heterogeneous small groups and used the same format as that used in the MMT condition, along with those metacognitive questions described in the preceding paragraphs. The metacognitive questions were used by each individual student when it was his or her turn to solve a problem aloud and by the group as a whole during the mathematical discourse. Also, the teacher used the metacognitive questions when she introduced the new
topic to the whole class, reviewed the lesson at the end of
the class, and provided help in the small groups.

**Control group:** In this condition, the control group served
as a comparison group with no intervention. Therefore, the
teachers of this group continued to teach as they usually did,
and students were not exposed to metacognitive training.
Each class started with a teacher’s short introduction of the
new concepts to the whole class. Then students practiced
the problems individually without using the metacognitive
questions. When students faced difficulties, they consulted
the students who sat near them. (In Israel, several students
sit at one desk, so they are used to asking for peers’ help
when they face difficulties.) The students discussed the
problem/task and provided help to each other until they
solved it. They asked for the teacher’s help only when they
did not solve their problem. At the end of the session, the
teacher reviewed the new concept with the whole class.

**Teacher Training and Learning Materials**

Prior to the beginning of this study, the six teachers par-
ticipated in a 1-day in-service training session that focused
on pedagogical issues related to teaching problem solving.
We told the teachers that they would be part of an experi-
ment in which new materials were being tested. They
worked with the new materials and learned how to use them
with their students. The materials included explicit lesson
plans, problems, and use of examples.

The teachers who were assigned to the IMPROVE method
were trained explicitly to use the metacognitive questions
with their students. We asked the teachers to model the use
of the metacognitive questions in their explanations and to
encourage their students to use the metacognitive questions
when they solved a problem. A discussion was held with the
MMT and UMT teachers on the use of metacognitive
processes in mathematics lessons. In addition, we trained the
MMT teachers to bridge the metacognitive processes used in
mathematics and English classrooms. The teachers of the
control group were not explicitly introduced to the meta-
cognitive technique. During the study, we observed all teachers
twice a week to ensure fidelity to the treatment.

**Measures**

We used three measures in this study to assess students’
mathematical reasoning and metacognitive knowledge: (a)
a pretest that focused on students’ mathematical knowledge
before the beginning of the study; (b) a posttest that assessed
students’ mathematical achievement, mathematical explana-
tions, and ability to transfer their knowledge to the solution
of a real-life task; and (c) a metacognitive questionnaire.

**Pretest of mathematics prior knowledge:** To control for
possible differences before the beginning of the study,
teachers administered a 41-item pretest to all students at the
beginning of the school year. The test covered arithmetic
knowledge taught prior to the beginning of the study in the
following content: whole numbers, fractions, decimals, and
percentages. The test was based on multiple-choice items
regarding basic factual knowledge and open-ended compu-
tation problems.

**Scoring:** For each item, students received a score of
either 1 (correct answer) or 0 (incorrect answer) and a total
score ranging from 0 to 41. The Kuder–Richardson reliabil-
ity coefficient was .87.

**Mathematics posttest:** A 72-item test assessed students’
mathematics achievement. The posttest covered the follow-
ing topics: rational numbers, identification of rational num-
bers on the number line, operations with positive and nega-
tive numbers, order of operations, and the basic laws of
mathematics operations. The test was composed of two kinds
of items: problems of both kinds were mixed. One kind (64
items) was based on multiple-choice items regarding basic
factual knowledge and open-ended computation problems;
the other kind (8 items) was specifically designed to assess
students’ mathematics reasoning. The 8 items included prob-
lems that ask students to draw conclusions about possible
outcomes, make algebraic generalizations, evaluate mathe-
matical expressions, and decide which mathematical laws are
appropriate for solving them. On those 8 items, students were
asked to give a final answer and to explain their reasoning in
writing. To gain a deeper understanding of students’ mathe-
imatics reasoning, we analyzed separately students’ mathe-
matics explanations provided for the 8 items.

**Scoring for the final answer:** For each item, students
received a score of either 1 (correct answer) or 0 (incorrect
answer) and a total score ranging from 0 to 72. The
Kuder–Richardson reliability coefficient was .87.

**Scoring for mathematical explanations:** A scoring proce-
dure was adapted from the holistic scoring rubrics developed
by Cai, Lane, and Jakabosin (1996). For each explanation,
students received a score between 0 and 2, and a total score
ranging from 0 to 16. For example, “In the following item, 
2³, ..., (-2)³, write the sign >, <, or = so that a correct state-
ment will be received. Explain your answer.” A score of 0
indicates incorrect explanations or explanations that are
irrelevant to the task (e.g., incorrect response: “2³ = [-2]³
because when there is a minus in brackets in powers, the
minus becomes +”). A score of 1 indicates an explanation
that has some satisfactory elements but may omit significant
parts of the problem (e.g., “2³ > [-2]³ because when the
exponent of the power is odd, the result will always be nega-
tive”—nothing was mentioned about 2³ or about the sign
>). A score of 2 indicates a correct response with a clear,
unambiguous explanation of one’s mathematical reasoning
(e.g., “2³ > [-2]³, when the exponent of the power is odd
and the base of the power is positive, the result is positive.
When the exponent of the power is odd and the base of the
power is negative, the result will be negative even with braces.
A positive number is always bigger than a negative number”).

Mathematical explanations were scored by two re-
searchers who we trained to use the holistic scoring rubrics
(Cai et al., 1996). The training included an introduction on
the principles of scoring by rubrics and practicing the model on some examples. First, each scorer rated the explanations independently. Then the scorer compared the scores and discussed the disagreements until consensus was achieved. The interjudge reliability for the first stage was .90.

The pizza task: In addition to the 72-item posttest described under the Mathematics posttest heading, students were administered the pizza task, which is based on a real-life situation. The task entails arranging refreshments for a party in school. Students received three different price proposals, and they had to check which was the most cost efficient and suitable to the limited budget at their disposal. Each student had to decide which offer was the most worthwhile, write a letter to the class treasurer in understandable language detailing his or her suggestion, and provide reasons for accepting the proposal. The pizza task is shown in the Appendix. The solution of the pizza task or any other kind of similar real-life problem was not studied in any of the classrooms.

Scoring: Students' answers were scored on four criteria, each ranging from 0 (no solution) to 5 (highest level solution); scores ranged from 0 to 20. The criteria were:

1. Referencing all data—referring to all the relevant data in each of the three proposals; price per pizza, price for supplements, diameter of the pizza, and the budget
2. Organizing the information—arranging the data in a table, diagram, algebraic expression, or any other representation for comparisons
3. Processing information—figuring the calculations correctly, describing explicitly the solution process
4. Drawing conclusions—presenting a proper solution to the required task, reporting the conclusions in a letter to the treasurer, giving reasons, justifying the choice, correctly referring to the data, and suggesting maximum exploitation of the money.

For examples of different levels of scoring, see the Appendix. Interjudge reliability for the four categories was .86.

Metacognitive questionnaire: The metacognitive questionnaire, adapted from the study of Montague and Bos (1990), assessed students' metacognitive knowledge regarding their specific and general problem-solving strategies. The questionnaire includes 25 items: 6 items refer to strategies used before the solution process (e.g., "Before I solve a problem in mathematics, I try to say it in my own words"); 5 items refer to strategies used during the solution process (e.g., "When I solve a problem in mathematics, I organize the data in a table"); 7 items refer to strategies used at the end of the solution process (e.g., "After I solve a problem, I check whether the answer is logical"); and 7 items refer to general problem-solving strategies used during cooperative learning (e.g., "When I talk about the problem with a friend, it's easier for me to understand it").

Scoring: Each item was constructed on a 5-point, Likert-type scale ranging from 1 (never) to 5 (always) and a total score ranging from 25 to 125. Cronbach's alpha reliability coefficient was .83.

Procedure

Within each school, teachers conducted classes according to their assigned teaching methods for the entire school year. In the present study, we focused on the problem-solving unit that was taught in all classrooms for about a month in the last semester. All students were administered the pretest at the beginning of the school year. Then, within each school, teachers taught as planned. The students were given the posttest, the pizza task, and the metacognitive questionnaire when they finished the unit.

Results

Mathematical Achievement

Our first purpose in this study was to investigate students' mathematical reasoning under the three conditions described in the Measures section. Table 1 reports the mean scores, adjusted mean scores, and standard deviations on the mathematical achievement and mathematical explanations, by time and treatment.

Results indicated that prior to the beginning of the study, we found no significant differences between the three conditions, F(2, 179) < 1.00, p > .05, on the mathematical pretest. One-way analyses of covariance on the posttest scores with the corresponding pretest scores used as a covariant indicated significant differences at the end of the study on mathematical achievement, F(2, 178) = 77.33, p < .0001, and on mathematical explanations, F(2, 178) = 98.87, p < .0001.

Effect sizes indicated that the MMT group outperformed the UMT group, which, in turn, outperformed the control group on mathematical achievement and mathematical explanations. (Effect sizes on mathematical achievement were .39 and .53 for comparing the MMT and UMT, and UMT and the control group, respectively. Effect sizes on mathematical explanations were .32 and 2.07 for comparing the MMT and UMT, and UMT and the control group, respectively.)

Post hoc analysis of the adjusted mean scores according to the pairwise comparison t-test technique (run on SPSS) indicated that the MMT group significantly (p < .02) outperformed the UMT group, which, in turn, significantly (p < .001) outperformed the control group on mathematical achievement. On mathematical explanations, however, the MMT and UMT groups outperformed the control group (p < .001). We found no significant differences between the MMT and UMT conditions.

The Real-Life Problem

Our second purpose in this study was to investigate the effects of the three conditions on students' ability to solve an authentic, real-life problem to which they were not exposed in the classroom. We performed a one-way multivariate analysis of covariance on the four criteria scores as dependent variables with the mathematical pretest scores
Table 1.—Mean Scores, Adjusted Mean Scores, and Standard Deviations on the Pretest, Posttest, and Mathematical Explanations, by Treatment

<table>
<thead>
<tr>
<th>Test</th>
<th>MMT (n = 60)</th>
<th>UMT (n = 60)</th>
<th>Control (n = 62)</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest: Prior knowledgea</td>
<td>72.83</td>
<td>69.73</td>
<td>73.03</td>
<td>&lt; 1.00</td>
</tr>
<tr>
<td>SD</td>
<td>15.60</td>
<td>15.91</td>
<td>19.90</td>
<td></td>
</tr>
<tr>
<td>Posttest: Mathematical achievementb</td>
<td>68.97</td>
<td>61.40</td>
<td>51.29</td>
<td>77.33****</td>
</tr>
<tr>
<td>M</td>
<td>11.10</td>
<td>14.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted M</td>
<td>68.25</td>
<td>63.01</td>
<td>50.43</td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>2.54</td>
<td>3.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Posttest: Mathematical explanationsc</td>
<td>13.37</td>
<td>12.15</td>
<td>4.11</td>
<td>98.87****</td>
</tr>
<tr>
<td>M</td>
<td>13.25</td>
<td>12.41</td>
<td>4.05</td>
<td></td>
</tr>
<tr>
<td>Adjusted M</td>
<td>2.54</td>
<td>3.11</td>
<td>3.88</td>
<td></td>
</tr>
</tbody>
</table>

Note. Adjusted mean scores refer to the posttreatment scores adjusted to the differences in the pretreatment scores. MMT = multilevel metacognitive training; UMT = unilevel metacognitive training.

Range: 0–100; a range: 0–72; b range: 0–16.

****p < .0001.

Table 2.—Mean Scores, Adjusted Mean Scores, and Standard Deviations on the Pizza Task, by Treatment

<table>
<thead>
<tr>
<th>Variable</th>
<th>MMT (n = 60)</th>
<th>UMT (n = 60)</th>
<th>Control (n = 62)</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>9.23</td>
<td>2.77</td>
<td>1.81</td>
<td>92.52****</td>
</tr>
<tr>
<td>M</td>
<td>9.14</td>
<td>2.97</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>Adjusted M</td>
<td>4.86</td>
<td>2.69</td>
<td>2.86</td>
<td></td>
</tr>
<tr>
<td>Reference to all data</td>
<td>2.57</td>
<td>0.83</td>
<td>0.58</td>
<td>95.22****</td>
</tr>
<tr>
<td>M</td>
<td>2.53</td>
<td>0.91</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>Adjusted M</td>
<td>1.18</td>
<td>1.06</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>Organizing information</td>
<td>1.30</td>
<td>0.37</td>
<td>0.06</td>
<td>34.76****</td>
</tr>
<tr>
<td>M</td>
<td>1.29</td>
<td>0.38</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Adjusted M</td>
<td>1.28</td>
<td>0.71</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>Processing information</td>
<td>2.06</td>
<td>0.17</td>
<td>0.32</td>
<td>61.52****</td>
</tr>
<tr>
<td>M</td>
<td>2.05</td>
<td>0.22</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>Adjusted M</td>
<td>1.56</td>
<td>0.64</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>Drawing conclusions</td>
<td>3.30</td>
<td>1.40</td>
<td>0.84</td>
<td>87.83****</td>
</tr>
<tr>
<td>M</td>
<td>3.27</td>
<td>1.47</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Adjusted M</td>
<td>1.48</td>
<td>0.80</td>
<td>1.17</td>
<td></td>
</tr>
</tbody>
</table>

Note. MMT = multilevel metacognitive training; UMT = unilevel metacognitive training.

****p < .0001.

used as a covariate. Findings indicated significant differences between the three learning conditions simultaneously, F(8, 350) = 28.12, p < .0001. The mean scores, adjusted mean scores, and standard deviations are reported on the pizza task by treatment (see Table 2).

Effect sizes indicated that the MMT group outperformed the UMT group on the total score (ES = 2.26) and on all the four criteria—referring all data (ES = 2.07), organizing information (ES = 3.88), processing information (ES = 2.20), and drawing conclusions (ES = 1.62). The UMT group further outperformed the control group on the total score (ES = 0.34) and on three criteria—referring all data (ES = 0.30), organizing information (ES = 1.29), and drawing conclusions (ES = 0.48). No significant differences
were found between the UMT and control group on processing information criteria (ES = 0.17).

Post hoc analysis of the adjusted mean scores according to the pairwise comparison t test technique (run by SPSS) indicated that the MMT group significantly (p < .001) outperformed the UMT group, which, in turn, significantly (p < .05) outperformed the control group on the total score and on three measures—referencing all data (p < .001, .02 for the comparison between the MMT and UMT, and UMT and the control group, respectively), organizing information (p < .001, .05 for the comparison between the MMT and UMT, and UMT and the control group, respectively), and drawing conclusions (p < .001, .01 for the comparison between MMT and UMT, and UMT and the control group, respectively). Regarding processing information, the MMT group outperformed (p < .001) the two other groups, but no significant differences were found between the UMT group and the control group.

**Metacognition**

Our third purpose in this study was to investigate differences in metacognitive knowledge among the three conditions. The mean scores and standard deviations on metacognitive measures by treatment are reported in Table 3. A one-way multivariate analysis of variance on the four criteria as dependent variables indicated significant differences between conditions on all metacognitive criteria simultaneously, F(8, 352) = 7.16, p < .0001.

Duncan post hoc analysis and effect sizes indicated that the MMT group significantly (p < .05) outperformed the UMT group, and the MMT group significantly outperformed the control group on the total score (ES = 0.84, 1.03) and on the four criteria—using problem-solving strategies before the solution process (ES = 0.44, 0.21), using problem-solving strategies during the solution process (ES = 0.26, 0.43), using problem-solving strategies at the end of the solution process (ES = 0.47, 0.48), and using problem-solving strategies during cooperative learning (ES = 0.83, 1.13). However, no significant differences were found between the UMT and the control group (ES = 0.19, 0.23, 0.18, 0.01, and 0.29, respectively, for the total score and the four criteria).

**Discussion and Conclusions**

The findings of the present study raise several questions: What is the role of metacognitive training in enhancing mathematical reasoning? Why did the MMT students outperform students in the UMT condition? Why were no significant differences found between UMT and the control group on metacognitive knowledge?

The present study indicates that training students to reflect on the similarities and differences between previous and new tasks, as well as on comprehending a problem before attempting a solution, and to consider the use of strategies that are appropriate for solving the problem enhances mathematical reasoning. Those findings are simi-

<table>
<thead>
<tr>
<th>Table 3.—Mean Scores and Standard Deviations on Metacognitive Measures, by Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Variable</strong></td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td><strong>M</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
</tr>
<tr>
<td>Problem-solving strategies used before solution process</td>
</tr>
<tr>
<td><strong>M</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
</tr>
<tr>
<td>Problem-solving strategies used during solution process</td>
</tr>
<tr>
<td><strong>M</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
</tr>
<tr>
<td>Problem-solving strategies used at end of solution process</td>
</tr>
<tr>
<td><strong>M</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
</tr>
<tr>
<td>Problem-solving strategies used during cooperative learning</td>
</tr>
<tr>
<td><strong>M</strong></td>
</tr>
<tr>
<td><strong>SD</strong></td>
</tr>
</tbody>
</table>

*Note. MMT = multilevel metacognitive training; UMT = unilevel metacognitive training.  
*p < .05. **p < .01. ***p < .001.
lar to previous studies (Chi, de Leeuw, Chiu, & Lavancher, 1994; Mevarech & Kramarski, in press; Slavin, 1996; Webb, 1989) that show that providing explanations is one of the best means for elaborating information and for making connections. By understanding why and how a certain solution to a problem has been reached, the student can elaborate on the information inherent in the explanations and can learn from it. Furthermore, evidence exists that metacognitive training facilitates metacognitive knowledge, which, in turn, affects mathematical reasoning and students’ ability to transfer their knowledge to new situations.

Masui and De Corte (1999) found that students who were exposed to metacognitive training had more knowledge about orienting and self-judging themselves than did students in the control groups. The metacognitive students also oriented themselves better when starting a new course. In this study, both meta-knowledge and transfer behavior were related positively to academic performance. Our findings show that only the MMT group significantly outperformed the other two groups on metacognitive knowledge. It is possible that our questionnaire was not sensitive enough to examine metacognitive processes in the MMT versus the UMT group. Further research based on systematic observations of the MMT and the UMT learning groups may identify other measures of metacognitive knowledge so that better evidence can be provided regarding the relation between metacognitive training and metacognitive knowledge.

**MMT Versus UMT**

Both MMT and UMT conditions were designed to use metacognitive processes that include comprehending problems, activating prior knowledge, and understanding which strategies are appropriate and for what reason. Students in MMT were taught to use metacognitive processes within mathematics and English classrooms and with regard to various contexts. The teachers guided MMT students to initiate metacognitive processes whenever they encountered a task—reading a text in a foreign language, solving a complex task, or even solving a conventional problem. In contrast, teachers taught UMT students to use metacognitive processes only within mathematics classrooms to solve conventional mathematics problems that require the application of ready-made algorithms.

According to constructivist theories, information is retained and understood through elaboration and construction of connections between prior knowledge and new knowledge (Wittrock, 1986). The likelihood of constructing networks of knowledge with MMT is greater than that with UMT. Those conclusions are similar to Cossey’s (1997) findings indicating that the more often seventh and eighth graders are exposed to metacognitive behaviors such as pattern seeking, conjectures, and giving reasons for ideas, the greater are their gains on mathematical reasoning. Similar conclusions also were reported by Kramarski and Mevarech (1997). The authors found that being exposed to metacognitive training implemented within a problem-solving-based Logo environment affects students’ ability to construct graphs and to reflect on their learning more than being exposed to a Logo-based problem-solving environment without explicit metacognitive training.

Furthermore, it is possible that because teachers trained MMT students to transfer their knowledge between foreign language (English) and mathematics language, students reflected more carefully on the solution of the transfer problem by considering all information, explaining their mathematical ideas, and writing an appropriate report of their conclusions. It also is possible that because they were trained to apply metacognitive processes in reading tasks and to solve mathematical problems, the MMT students approached a real-life problem as a story embedded within quantitative information. Therefore, they were better able to transfer their knowledge to a new situation.

Our findings support earlier conclusions (Hoek, van den Eden, & Terwel, 1999; Mevarech, 1999) that metacognitive training is effective for developing problem-solving ability because it enables one to link quantitative knowledge and situational knowledge. When the two types of knowledge are joined, a mental representation is constructed that supports mathematical reasoning (Cecil & Roazzi, 1994).

**Implications for Future Research**

In our study, we examined the effects of MMT on mathematical reasoning. We suggest that researchers similarly examine the different effects of MMT and UMT when UMT is implemented in English classrooms. Furthermore, to be able to generalize our findings, MMT and UMT approaches should be used on a larger scale and for a longer period of time than in this study. We assume that such an implementation could be realized if schools would be transformed into “smart schools” (Perkins, 1992) in which teachers and students reach the highest levels of awareness of cognitive, affective, and regulative processes (Masui & De Corte, 1999). Given the findings and suggestions of the present study, there is a need to develop metacognitive training methods that are appropriate for various domain-specific areas in mathematics as well as in other subjects. The issue of the conditions under which each method works best merits future research.

An interesting question raised in this study relates to the different role of multilevel versus double portion metacognitive training on achievement. To address the issue, students who are exposed to metacognitive training in one classroom should be compared with students who are exposed to metacognitive training in one classroom, for twice as long. One also may want to compare the effects of metacognitive training implemented in mathematics classrooms with metacognitive training implemented in reading comprehension or other classrooms.
## APPENDIX
### The Pizza Task

Your classmates organize a party. The school will provide the soft drinks, and you are asked to order the pizza. The class budget is 85 N.I.S. and you want to order as many pizzas as you can. Here are proposals of 3 local pizza restaurants and their prices. Compare the prices and suggest the cheapest offer to the class treasurer.

<table>
<thead>
<tr>
<th>Type of pizza</th>
<th>Price per pizza (in N.I.S.)</th>
<th>Diameter</th>
<th>Price for supplements (in N.I.S.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pizza Boom</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Personal</td>
<td>3.50</td>
<td>15</td>
<td>4.00</td>
</tr>
<tr>
<td>Small</td>
<td>6.50</td>
<td>23</td>
<td>7.75</td>
</tr>
<tr>
<td>Medium</td>
<td>9.50</td>
<td>30</td>
<td>11.00</td>
</tr>
<tr>
<td>Large</td>
<td>12.50</td>
<td>38</td>
<td>14.45</td>
</tr>
<tr>
<td>Extra large</td>
<td>15.50</td>
<td>45</td>
<td>17.75</td>
</tr>
<tr>
<td>Super Pizza</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>8.65</td>
<td>30</td>
<td>9.95</td>
</tr>
<tr>
<td>Medium</td>
<td>9.65</td>
<td>35</td>
<td>10.95</td>
</tr>
<tr>
<td>Large</td>
<td>11.65</td>
<td>40</td>
<td>12.95</td>
</tr>
<tr>
<td>McPizza</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>6.95</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>Large</td>
<td>9.95</td>
<td>35</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Note. N.I.S. = new Israeli shekels.

**Example for Scoring:**

I think that we have to buy a large pizza of the "Super Pizza." We can buy 7 super pizzas and it will cost us 81.55 N.I.S. I will divide every pizza into 8 pieces and it will be enough for 56 children. We will not order any supplements. We will be left 3.45 N.I.S. I think that we should buy that pizza.

**Scoring:**

1. **Reference to all data:** The student referred to only one of the proposals—Score 1.
2. **Organizing the information:** The student did not use any representation to present his calculation or conclusion—Score 0.
3. **Processing information:** The calculations are correct but the student did not write explicitly the solution process—Score 3.
4. **Drawing conclusions:** The student explained his choice, but he did not compare his conclusion to other results and he did not justify his reasoning (e.g., we will not order supplements)—Score 3.

**Total score: 7**

### Super Pizza

<table>
<thead>
<tr>
<th>Type of pizza</th>
<th>Number of pizzas</th>
<th>Total number of personal pieces</th>
<th>Price per pizza (in N.I.S.)</th>
<th>Price to pay (in N.I.S.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 small</td>
<td>85:65–10</td>
<td>10 × 4 = 40</td>
<td>8.65</td>
<td>86.5</td>
</tr>
<tr>
<td>6 medium</td>
<td>85:9.65–9</td>
<td>9 × 6 = 54</td>
<td>9.65</td>
<td>86.85</td>
</tr>
<tr>
<td>8 large</td>
<td>85:11.65–7</td>
<td>7 × 8 = 56</td>
<td>11.65</td>
<td>81.55</td>
</tr>
</tbody>
</table>

### McPizza

<table>
<thead>
<tr>
<th>Type of pizza</th>
<th>Number of pizzas</th>
<th>Total number of personal pieces</th>
<th>Price per pizza (in N.I.S.)</th>
<th>Price to pay (in N.I.S.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 small</td>
<td>85:6.95–12</td>
<td>12 × 4 = 48</td>
<td>6.95</td>
<td>83.4</td>
</tr>
<tr>
<td>8 large</td>
<td>85:9.95–8</td>
<td>8 × 8 = 64</td>
<td>9.95</td>
<td>79.6</td>
</tr>
</tbody>
</table>

**Dear Class Treasurer,**

The results of my calculations:

The best pizza for us to buy is from McPizza. We can buy 8 large pizzas (it means big diameter), and in each one 8 pieces. We get 64 pieces and have to pay only 79.6 N.I.S. There is money left over for a drink. It is great!

### Pizza Boom

<table>
<thead>
<tr>
<th>Type of pizza</th>
<th>Number of pizzas</th>
<th>Total number of personal pieces*</th>
<th>Price per pizza (in N.I.S.)</th>
<th>Price to pay (in N.I.S.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Personal</td>
<td>85:3.5–24</td>
<td>24</td>
<td>3.5</td>
<td>84</td>
</tr>
<tr>
<td>Small, 4 pieces per pizza</td>
<td>85:6.5–13</td>
<td>13 × 4 = 52</td>
<td>6.5</td>
<td>84.5</td>
</tr>
<tr>
<td>Medium, 6 pieces per pizza</td>
<td>85:9.5–9</td>
<td>9 × 6 = 54</td>
<td>9.5</td>
<td>85.5</td>
</tr>
<tr>
<td>Large, 8 pieces per pizza</td>
<td>85:12.5–7</td>
<td>7 × 8 = 56</td>
<td>12.5</td>
<td>87.5</td>
</tr>
<tr>
<td>Extra large, 10 pieces per pizza</td>
<td>85:15.5–5</td>
<td>5 × 10 = 50</td>
<td>16.5</td>
<td>77.5</td>
</tr>
</tbody>
</table>

*I calculated the number of pizzas I could buy for 85 N.I.S. and multiplied the results with the number of pieces we have in each pizza, and I got the total number of personal pieces.

**Scoring:**

1. **Reference to all data:** The student referred to the class budget (85 N.I.S.) and to the data of the three proposals. But he did not refer to the diameter of the pizza and the price for supplements—Score 3.
2. **Organizing the information:** The student used various representations: table, diagrams of pie and verbal explanations—Score 4.
3. **Processing information:** The calculations are correct but the student wrote clearly the solution process—Score 5.
4. **Drawing conclusions:** The student explained his choice and justified his reasoning—Score 5.

**Total score: 17**
REFERENCES


Although cancer is a very grown-up disease, thousands of children like Adam learn all about it each year when they’re diagnosed with one of its deadly forms.

But these children have a fighting chance because of life-saving research and treatments developed at St. Jude Children’s Research Hospital. To learn more about the work doctors and scientists are doing at St. Jude and how you can help, call:

1-800-877-5833.

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Danny Thomas, Founder