NO SCRATCH PAPER – TIME LIMIT TWO HOURS

The point value of each problem is in the left-hand margin. You must show your work to receive full credit for your answers, except on problems 1. Round off all decimal answers to two places. Work neatly.

(30) 1. Fill in the blank.

(a) In a standard triangle, \( a^2 = b^2 + c^2 \) according to the Law of Cosines.
(b) As \( x \to -\infty \), \( f(x) = -x^3 + 10x^2 - 1 \to \).
(c) The general term of the sequence 2, 5, 9, 14, 20, \( \cdots \) is \( a_n = \) for \( n = 1, 2, 3, \cdots \).
(d) The exact value of the angle \( \cos^{-1}(\cos \frac{7\pi}{6}) \) is radians.
(e) If \( f \) is a one-to-one function and \( f(2) = -7 \) and \( f(\frac{1}{7}) = 2 \), then \( f^{-1}(2) = \).
(f) The horizontal asymptote of the graph of the function \( h(x) = \frac{12x^2+5}{3x^2-7} \) is the line .
(g) The exact value of \( \log_4 4^{-3} \) is .
(h) The phase shift of the function \( f(x) = 3\cos(2x - \pi) \) is .
(i) 175 degrees is radians.
(j) The graph of the function \( y = 2x^5 - 7x^3 + 2x - 1 \) has at most turning points.
(k) The point \( (r, \theta) = (-2, 30^\circ) \) in rectangular coordinates is \( (x, y) = (\quad, \quad) \).
(l) The domain of the function \( f(x) = \frac{1}{\sqrt{x+3}} \) is \( (\quad, \quad) \).
(m) The graph of the function \( g(x) = \ln(x-1)+3 \) is the graph of the function \( f(x) = \ln x \) shifted to the one unit and shifted up units.
(n) \[ \begin{bmatrix} -3 \\ 2 \end{bmatrix} \begin{bmatrix} 4 & 1 \end{bmatrix} = \].
(o) The solution interval of the absolute value inequality \( |x| < 3 \) is \( (\quad, \quad) \).
(p) The graph of the polar function \( r = 2\cos \theta \) is a .
(q) \( -4(-2\vec{r} + \vec{j}) + 3(\vec{r} - 5\vec{j}) = \).
(r) The sequence 8, 4, 2, 1, \( \frac{1}{2} \), \( \cdots \) is a sequence.
(s) If \( 2^{-x} = 8 \), then \( x = \).
(t) The magnitude of the vector \( \vec{v} = 2\vec{r} - 3\vec{j} \) is .

\[1\text{If you exceed the time limit you will receive a grade of zero.} \] However, you are allotted an extra 5 minutes for checking in and out.
(15) 2. The graph of four of the following functions are shown below. Find the function corresponding to each graph. To receive partial credit, state your reasons.

(a) \( r = 2 \sin \theta \)  
(b) \( y = x(x + 2)(x - 1) \)  
(c) \( y = \cos x + 1 \)  
(d) \( y = x^2(x + 2) \)

(e) \( y = \sqrt{x + 1} \)  
(f) \( y = -\sin x \)  
(g) \( y = 3(x - 1)^2 + 2 \)  
(h) \( \theta = \pi \)

(i) \( y = e^x - 1 \)  
(j) \( y = -3x^2 + 6x + 1 \)  
(k) \( y = \ln(x + 1) \)  
(l) \( r = 1 + \sin \theta \)

![Graphs](image)

Graph of _____ ; Graph of _____ ; Graph of _____ ; Graph of _____

(10) 3. How long will it take for a $10,000 investment in an account with 6% interest rate compounded monthly to grow to $20,000?

(10) 4. Find the inverse of the 1-1 function \( f(x) = 2^x \).
5. Solve the following equations and inequality algebraically.

(a) Solve \( \log_{12}(x + 5) + \log_{12}(x + 1) = 1 \).

(b) \( x^3 - x^2 - 6x > 0 \) State the answer in interval notation.

(c) \( \cos 2\theta + 3 = 5 \cos \theta \) State the answer in degrees.

6. Find the point(s) of intersection of the graph of the parabola \( y = x^2 - 2x + 1 \) and the line \( y + x = 1 \).
(15) 7. Find the $x$- and $y$-intercepts, horizontal and vertical asymptotes of the graph of the function 
\[ f(x) = \frac{x^2 + 4}{2(x-1)(x+1)} \]. Use at least 6 points to graph it.

(15) 8. Draw the graph of the function 
\[ g(x) = \begin{cases} \sin x & \text{for } x < 0 \\ \lfloor x \rfloor & \text{for } x \geq 0 \end{cases} \] 
by using at least 6 points. Note: 
\([x] = \text{int}(x)\) is the greatest integer less than or equal to \(x\).
9. Find the exact value of $\cos 15^\circ$.

10. The angles of elevation from two points on the opposite sides of a tree to the top of that tree are $30^\circ$ and $25^\circ$. If the total distance between these two points is 100 feet, what is the height of the tree?

11. Find a standard equation of a hyperbola with foci $(\pm 5, 0)$ and vertices $(\pm 3, 0)$. State its asymptotes.
(15) 12. Solve the system \[ \begin{align*} 2x + 3y &= -1 \\ 3x + 5y &= 0 \end{align*} \] using the inverse matrix method.

(15) 13. Find the sum of the following series.

(a) \[ 1 + 4 + 7 + 10 + \cdots + 61 \]

(b) \[ 3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots \]